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Holdup propagation predicted by steady-state drift flux models

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Abstract

For actual two phase pipe systems, variations in input flows often occur naturally, or may be imposed intentionally. Changes in holdup (in-situ fluid fraction) caused by such input variation will propagate along the pipe, and may affect overall performance. The current paper describes experimental investigation of holdup propagation in oil–water flow in a vertical pipe. It is shown that larger changes in input holdup profile may either compress or rarefy as they propagate along the pipe. For the cases considered, observed behaviour could be quite accurately predicted using a non-linear, hyperbolic wave propagation relation and a drift flux model, calibrated at steady-state flow conditions. The methodology outlined enables prediction and optimization of processes where the fluid content in a pipeline, or wellbore, is displaced by another, immiscible fluid.

Keywords: Vertical oil–water flow; Kinematic waves; Measurement and prediction

1. Introduction

The current work was spawned by a specific practical problem: the "bull-heading" of oil and gas production wells. In bull-heading procedure, water is pumped against well pressure through a tubing head valve, to displace wellbore fluids down the tubing and back into the reservoir. If everything goes well, a water-filled wellbore results, in hydrostatic equilibrium with the reservoir. However, some mixing always occurs between the wellbore fluid and the water displacing it. This may leave hydrocarbons in the wellbore and prevent hydrostatic equilibrium after the pumps have stopped. To properly plan and perform bull-heading, it is necessary to predict the length of the mixing zone and the hydrocarbons contained within it.

It appeared reasonable to consider mixing zone development as propagation of the initial step-change in water holdup (water fraction) imposed by pumping water into the tubing head. Thus, the mixing zone development could be described by wave propagation principles. This

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idea was accepted by the operating company (NAM) and a field test performed based on this and other pertinent considerations. The field test was successful and the result was published by Oudeman et al. (1994).

However, although the practical problem outlined above could be considered solved, it was felt that the field test alone did not completely verify the basic assumptions on which it had been designed. Also, it was felt that the displacement of immiscible fluids might have broader applications. It was therefore decided to continue the investigation in a simplified laboratory setting, to study basic properties of holdup propagation.

Considering the basis of holdup propagation, an early treatise was offered by Wallis (1969). Since the propagation behaviour can be estimated from the continuity relations, he used the term "continuity wave". The terms "void fraction waves" and "kinematic waves" have been used by other researchers to describe similar phenomena.

Matuszkiewicz et al. (1987) measured the propagation of small, spontaneous disturbances in nitrogen–water flow in a vertical, square test section. They found that at sufficiently high gas concentration, low-frequency disturbances amplify along the pipe. This was associated with the transition from bubble to slug flow. Saiz-Jabardo and Bouré (1989) extended and supplemented the above-mentioned work with measurements on imposed, cyclic variations in a circular small diameter (25 mm i.d.) pipe. They suggest that slugging may develop when the propagation speed of a given disturbance is higher than the gas phase velocity. Kytömaa and Brennen (1991) studied the attenuation and propagation of naturally occurring continuity waves in a larger diameter (100 mm i.d.) pipe. They investigated both gas–liquid and solid–liquid flow, arriving at conclusions qualitatively similar to those above.

Mathematical models of holdup propagation in bubbly flow have been developed by Pauchon and Banerjee (1995). Pauchon (1989) extends the model by Pauchon and Banerjee (1986) and shows it applicability to stability analyses of horizontal, stratified two-phase flow. Biesheuvel and Gorissen (1990) show that propagation at low frequency can be approximated by a linearized Burgers/Korteweg-de-Vries equation. This includes some dispersivity, whereas earlier models predict non-dispersive propagation. Park et al. (1990b) derive a linearized holdup propagation model, which also predicts void wave dispersion.

Most older experimental works report non-dispersive behaviour. However, Matuszkiewicz et al. (1987) observe that continuity waves may be slightly dispersive. Bouré (1988) presents data analyses revealing the excistence of two kinematic modes with different velocities and dampening. He also proposes a dynamic extension of the drift flux model concept to better represent observed behaviour.

The works mentioned above mainly consider propagation of small variations, with the primary goal of investigating flow regime stability and transition. The current work considers large, imposed variation of input holdup, with the primary objective of investigating the displacement of a fluid by another, immiscible one. The model is developed based on a simple non-linear wave equation, solved by the method of characteristics. Assuming that local flow behaviour is governed by steady-state slip flow mechanisms, propagation phenomena are sought predicted based on steady-state holdup relations. A non-linear wave propagation model, based on solution of the two-phase, simultaneous flow relation, has been developed by Park et al. (1990a). They used interfacial shear to represent slip between the flowing phases. However, information on interfacial shear is less available than holdup data.

So, for practical applications, the current state of knowledge may favour models based on holdup correlations.

Experimental work was carried out with oil and water flowing in a vertical pipe. Holdup was recorded by impedance cells at two locations. This enabled measurement of propagation velocity and profile distortion. The measured results and predictions were quite consistent, as shown below.

2. Prediction of holdup propagation

2.1. Wave model

Transient modelling requires consideration of continuity, in addition to the flow and closure relations normally used to describe steady-state flow. Considering slow variations, the flow may be considered incompressible. The unidirectional continuity requirements for two immiscible and incompressible fluids, say oil and water, can be written as:

$$\frac{\partial y_o}{\partial t} + \frac{\partial v_{so}}{\partial x} = 0 \tag{1}$$

$$\frac{\partial y_w}{\partial t} + \frac{\partial v_{sw}}{\partial x} = 0 \tag{2}$$

$$y_o + y_w = 1 \tag{3}$$

where y_o = oil holdup, volume fraction occupied by oil; y_w = water holdup, volume fraction occupied by water; v_{so} = oil superficial velocity (oil rate divided by total flow area); v_{sw} = water superficial velocity (water rate divided by total flow area).

From the continuity relations above, it follows that oil and water superficial velocities add up to a superficial mixture velocity, v_m, constant at a given time

$$v_{so} + v_{sw} = v_m. \tag{4}$$

Thus, although the oil and the water flows may vary along the pipe, their sum is constant. The continuity relations above may be transformed to a wave equation, describing translation of holdup changes

$$\frac{\partial y_w}{\partial t} + v_c \frac{\partial y_w}{\partial x} = 0 \tag{5}$$

$$v_c = \left.\frac{dv_{sw}}{dy_w}\right|_{v_m} \tag{6}$$

Steady-state holdup is usually expressed as a function of superficial velocities, $y_w(v_{so}, v_{sw})$. Equivalently, the water superficial velocity may be expressed as a function of water holdup and mixture velocity

$$v_{sw} = v_{sw}(y_w, v_m). \tag{7}$$

Using the relations above, (6) and (7), the characteristic velocity can be calculated for a given holdup model or slip flow relation. It thereby follows that the characteristic velocity will be a function of local water holdup and mixture velocity: $v_c = v_c(y_w, v_m)$.

2.1.1. Linear propagation

Consider a small change in holdup. The characteristic velocity may be considered a constant determined by the steady-state holdup

$$\bar{v}_c = v_c(\bar{y}_w, v_m). \tag{8}$$

Thus (5) can be written as

$$\frac{\partial y_w}{\partial t} + \bar{v}_c \frac{\partial y_w}{\partial x} = 0. \tag{9}$$

Equation (9) above describes linear wave propagation and has the following simple solution:

$$y_w = y_w(x - \bar{v}_c t). \tag{10}$$

Physically, (10) implies that small variation in input holdup will propagate along the pipe at a constant characteristic velocity. This is often referred to as the continuity wave velocity (Fitreman and Verdrines, 1985; Wallis, 1969).

2.1.2. Non-linear propagation

For larger changes in holdup, the characteristic velocity can no longer be assumed a constant. To investigate wave propagation, consider first the total differential of water holdup

$$\frac{dy_w}{dt} = \frac{\partial y_w}{\partial t} + \frac{\partial y_w}{\partial x}\frac{dx}{dt}. \tag{11}$$

Consider fixed water holdup:$dy_w/dt = 0$. By comparing (5) and (11) it follows that this will propagate at velocity v_c.

$$\frac{dx}{dt} = v_c(y_w, v_m). \tag{12}$$

The propagation velocity of any fixed level of holdup can be computed according to (6). Since each holdup level propagates at its characteristic velocity, future holdup profiles may be predicted by propagation of the initial profile. Depending on the input profile and the characteristic velocity as a function of holdup, the profile may rarefy, or compress. A compressing profile will ultimately develop into a shock (Whitham, 1973). The propagation and distortion of holdup profiles, or waves, can be predicted by the theory above, considering any specific slip flow model, or holdup correlation. However, different slip flow models may predict different propagation and distortion behaviour. This is shown by two examples below.

2.2. Holdup propagation velocity predicted by the original drift flux model

The original drift flux model was proposed by Zuber and Findlay (1965). It assumes a linear relation between the apparant velocity of the lighter phase (oil in our case) and the total superficial velocity

$$v_o = \frac{v_{so}}{1 - y_w} = C_o v_m + v_\infty. \tag{13}$$

The dynamic velocity ratio above, C_o, may be interpreted as a correction factor for uneven distribution of the discontinous phase. In the case of a homogenous mixture, C_o should equal unity. The drift velocity, v_∞, may for oil droplets in water be interpreted as the rise velocity due to buoyancy. From (13) above, the water superficial velocity can be expressed as a function of water holdup and total superficial velocity

$$v_{sw} = (C_o v_m + v_\infty) y_w - (C_o - 1) v_m - v_\infty \tag{14}$$

The characteristic velocity, as defined by (6), can now be derived by differentiation of (14):

$$v_c = C_o v_m + v_\infty. \tag{15}$$

For a given total superficial velocity, (15) predicts constant characteristic velocity. Thus, the original drift flux model predicts that any holdup wave, or profile, will propagate undistorted along the pipe, at constant characteristic velocity.

2.3. Holdup propagation velocity predicted by a modified drift flux model

A modified drift flux model was used by Asheim (1986) to analyse production data from oil and gas wells. An equivalent approach was used by Hill (1992), to interpret oil and water flow measurements. The model is based on an assumed linear relation between the velocity of the lighter phase (oil) and the velocity of the denser phase (water)

$$v_o = C_o v_w + v_\infty. \tag{16}$$

The physical interpretation of (16) is broadly similar to that of the original drift flux model, outlined above. However, in the original drift flux, (13), the dynamic velocity ratio, C_o, refers to mixture flow velocity. In the modified drift flux model, (16), the dynamic velocity ratio refers to velocity of the water surrounding oil droplets. This may appear more logical, since droplets are in direct contact with the surrouding fluid. This difference generally provides more realistic prediction of slip behaviour when the flow rate is low. Comparing (13) and (16), it is evident that the parameters, C_o, v_∞, are not equivalent to similar parameters in the original drift flux model. However, to simplify the notation, the same indices will be used. Defining flow velocity as the ratio of superficial velocity and water holdup, the water superficial velocity can be derived from (16) above.

$$v_{sw} = \frac{(v_m - v_\infty) y_w + v_\infty y_w^2}{C_o(1 - y_w) + y_w}. \tag{17}$$

The holdup propagation velocity is derived by differentiation of (17) above. This gives

$$v_c = \frac{(v_m - v_\infty) C_o + 2 C_o v_\infty y_w - (C_o - 1) v_\infty y_w^2}{(C_o(1 - y_w) + y_w)^2}. \tag{18}$$

According to the result above, the characteristic velocity depends on water holdup. Thus, a holdup profile, generated for example by changes in input flowrates, may distort as it propagates along the pipe. If the characteristic velocity increases with increasing water holdup ($dv_c/dy_w > 0$), then an increasing holdup profile will compress as it propagates along the pipe. By differentiation of (18), we find the following general conditions for characteristic velocity increasing with increasing holdup:

$$v_m > \frac{-v_\infty}{C_o - 1}; \quad \text{if } C_o > 1 \tag{19}$$

or

$$v_m < \frac{v_\infty}{1 - C_o}; \quad \text{if } 0 < C_o < 1. \tag{20}$$

Thus, if (19) (or alternatively (20)) is fulfilled, a gradual increase in input holdup will compress as the change propagates along the pipe. Conversly, a decreasing holdup profile will rarefy. Equations (19) and (20) above provide simple criteria. However, their main significance is that rarefication, or compression, is independent of holdup. This is not obvious considering the characteristic velocity relation, (18), from which the criteria above are derived.

3. Measured holdup propagation

3.1. Experimental facility

The derivations above show that superficially similar slip flow models predict propagation quite differently. Actual propagation behaviour was investigated experimentally, with light oil and water flowing through a vertical PVC-pipe of inner diameter 42.6 mm and length 7.80 m. The experimental facility is illustrated by Fig. 1. The fluid parameters are listed in Table 1.

The input flow rates of oil and water were measured by precision rotameters. Local water holdup was measured at the locations indicated in Fig. 1, using impedance cells. Fig. 2 shows a cross-section through the impedance cell used. These cells are similar to those described by Andreussi et al. (1988), although somewhat simpler.

The impedance cells enable measurement of conductance and capacitance of the flowing mixture, using high-frequency, alternating current. For the experiments described below, only conductance (real component) was measured. The water was salted slightly to obtain distinct conductance contrast against the oil. The impedance cells provide very fast response. After initial calibration at steady-state conditions, they were used to measure change in holdup with time. This enabled measurement of holdup changes with time and interpretation of holdup profiles passing through the locations of measurement.

The flow pattern in the experiment described below was a continous water phase, containing oil in various concentrations. Bubbly flow was observed at virtually all conditions. Emulsions tended to develop at rates higher than those considered below.

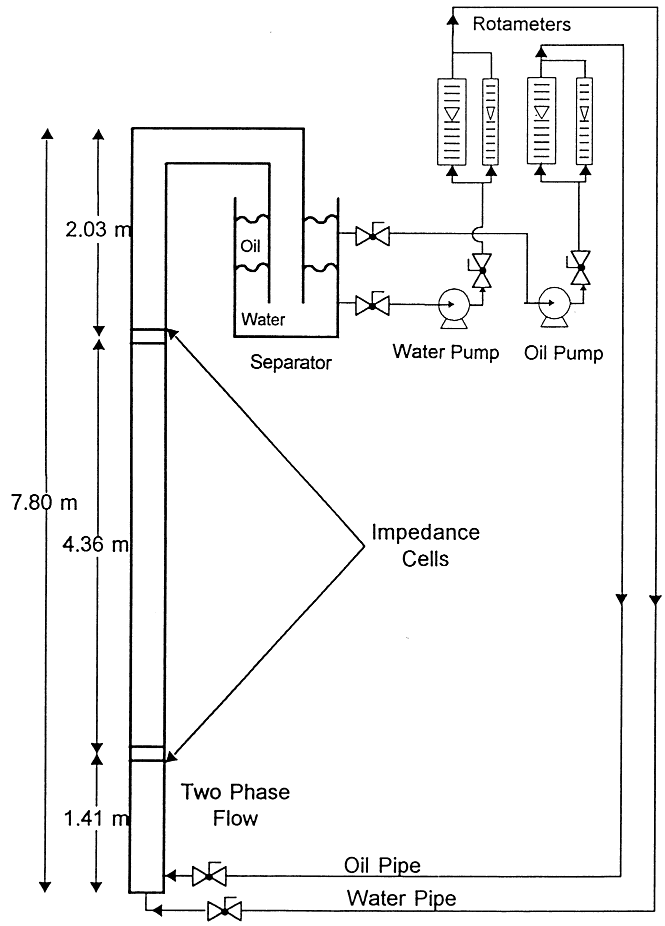

Fig. 1. Experimental facility.

3.2. Steady-state calibration

The impedance cells were calibrated under steady-state flow conditions. The steady-state holdup was measured by closing the inlet and outlet valves, and metering the total oil and water volumes in the pipe after segregation. Fig. 3 shows the calibration curve for the upper cell. For the curve illustrated, the total superficial velocity has been maintained constant at 0.20 m/s. However, the calibration curves for other total superficial velocities were similar.

Table 1
Fluid data

Water density	1000 kg/m^3
Water viscosity	1.2 cp
Oil density	783 kg/m^3
Oil viscosity	1.4 cp

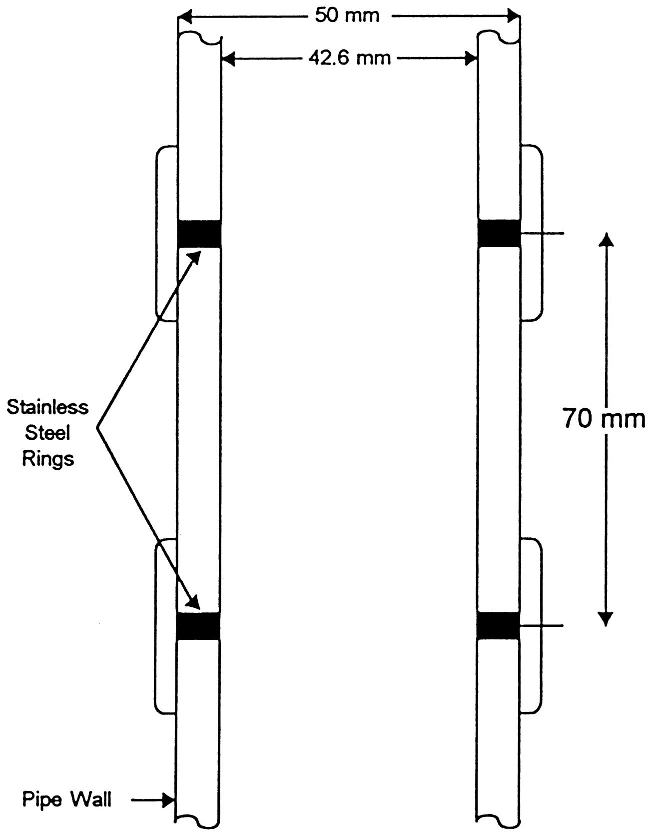

Fig. 2. Cross section of impedance cell.

Thus, the signal measured seemed insensitive to flow velocity and thereby insensitive to droplet size. As observed from Fig. 3, the calibration curve is linear in the holdup range corresponding to continuous water phase.

3.2.1. Drift flux relationship

A total of 60 steady-state tests were performed to establish empirical drift-flux relations describing oil–water flow in the experimental facility. In the tests five different total superficial velocities were considered (Table 2). All tests were performed in the bubble flow regime, with water as continuous phase.

Fig. 4 compares measured water holdup and flux fraction (no-slip holdup). At low overall velocity, Group A data (ref. Table 2) the measured water holdup is much higher than the flux fraction. This indicates considerable water loading, due to slippage, at the low overall velocity.

Fig. 5 shows measured results plotted according to the original drift flux model, (13). For flow behaviour according to the original drift flux model, all points in Fig. 5 should plot along the same straight line. By linear regression such a straight line may be described by the parameters C_o = 0.62 and v_∞ = 0.094 m/s. Hill (1992) estimates parameter values for the original drift flux model: C_o=0.61 and v_∞=16.69 ft/min = 0.085 m/s, for oil–water flow at

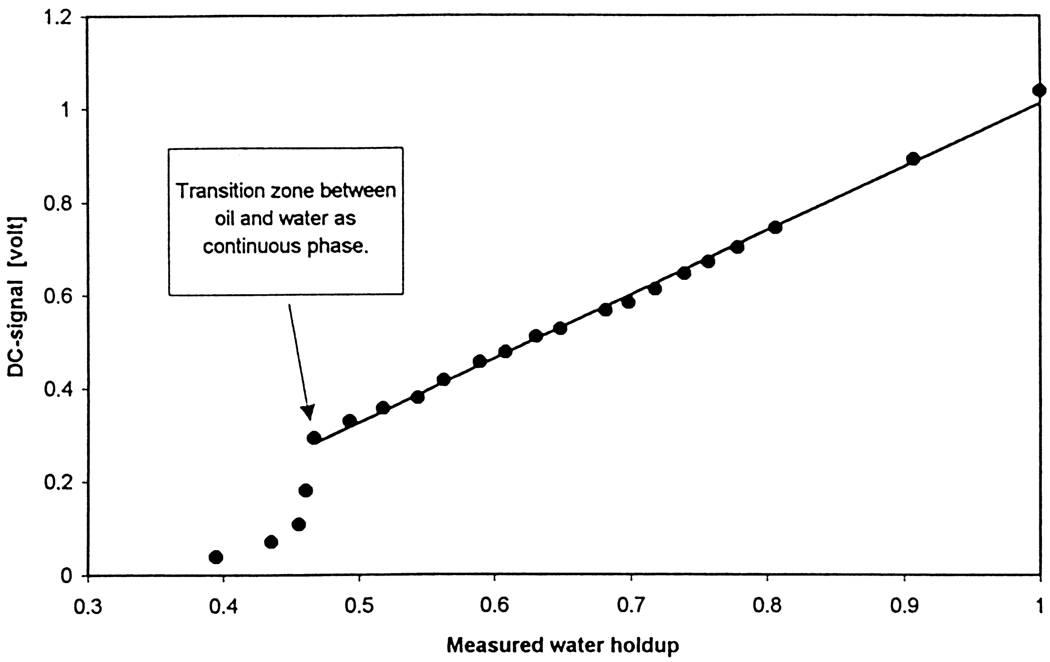

Fig. 3. Calibration curve for impedance cell.

high water holdup. Such parameter values may be physically meaningful. However, as observed from the experiments, the precision at lower flow rate is poor. Fig. 6 shows measured results plotted according to the modified drift flux model, [16]. Predicting the modified drift flux parameters in the traditional way, by linear regression of all tests together, yields $C_o = 0.58$ and $v_\infty = 0.11$ m/s. This is obviously not very precise in the lower velocity range. From Fig. 6 it is observed that the lower flow rates, Groups A–C, plot as separate straight lines. The corresponding model parameters, C_o and v_∞, estimated by linear regression, are listed in Table 2. The data in Group D show different flux behaviour. Below water holdup of 0.72 these data can be described by $C_o = 3.74$ and $v_\infty = -0.20$ m/s, while above the data seem to be described by $C_o = 0$ and $v_\infty = 0.182$ m/s.

Table 2
Flow test data

Group	A	B	C	D	E
Superficial velocity (m/s)	0.050	0.075	0.100	0.125	0.200
Water flux fraction range	0.36-0.60	0.25-0.73	0.25-0.80	0.36-0.80	0.40-0.90
Reynolds number range*	1380-1520	1980-2400	2640-3290	3450-4110	5610-6830
Parameters in the modified drift flux model (by linear regression)					
Dynamic velocity ratio: C_o	4.92	3.46	2.81	—	—
Drift velocity (m/s): v_∞	−0.0215	-0.0416	−0.0561	—	—

* Mixture properties are calculated as weighted averages of the individual phase properties with input fraction as weighting factor.

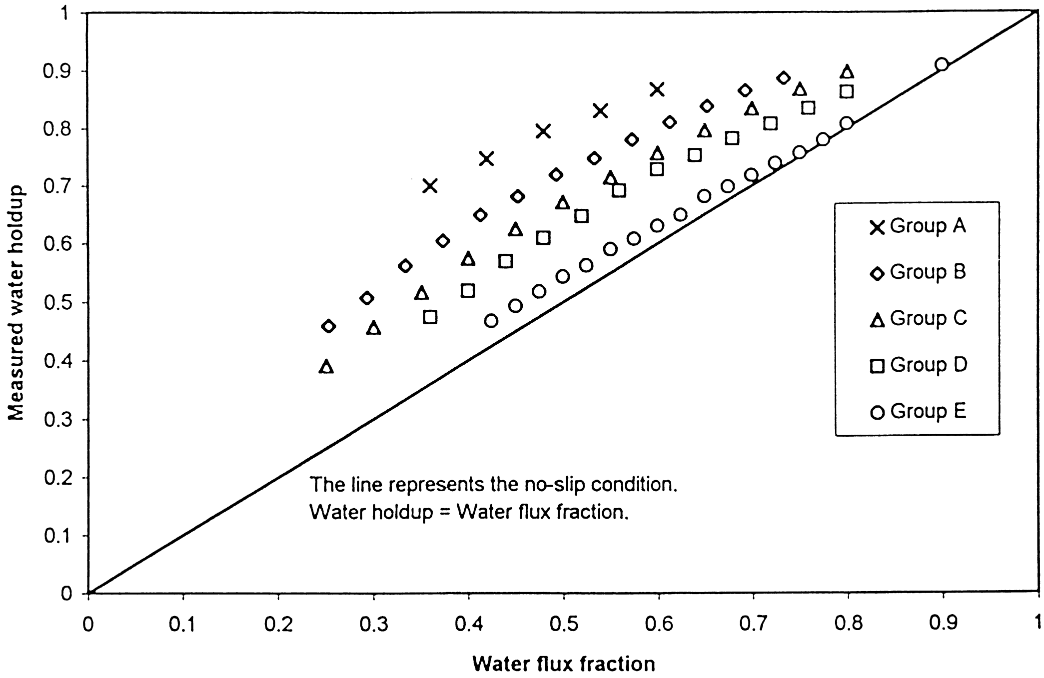

Fig. 4. Measured water holdup vs water flux fraction, for all test groups.

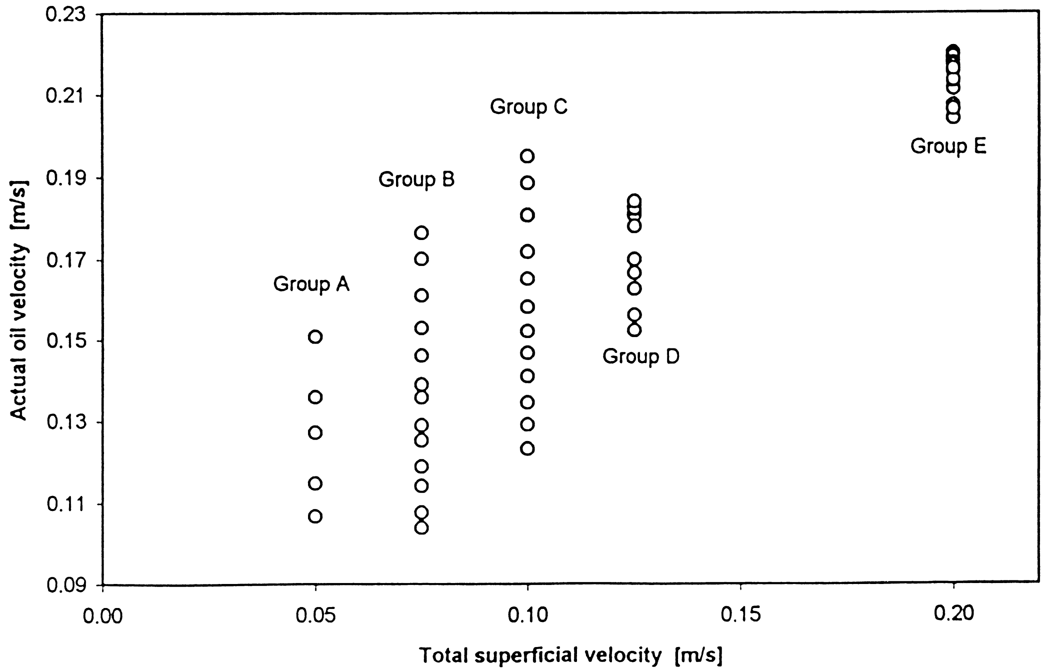

Fig. 5. Measured results, plotted according to the original drift flux model.

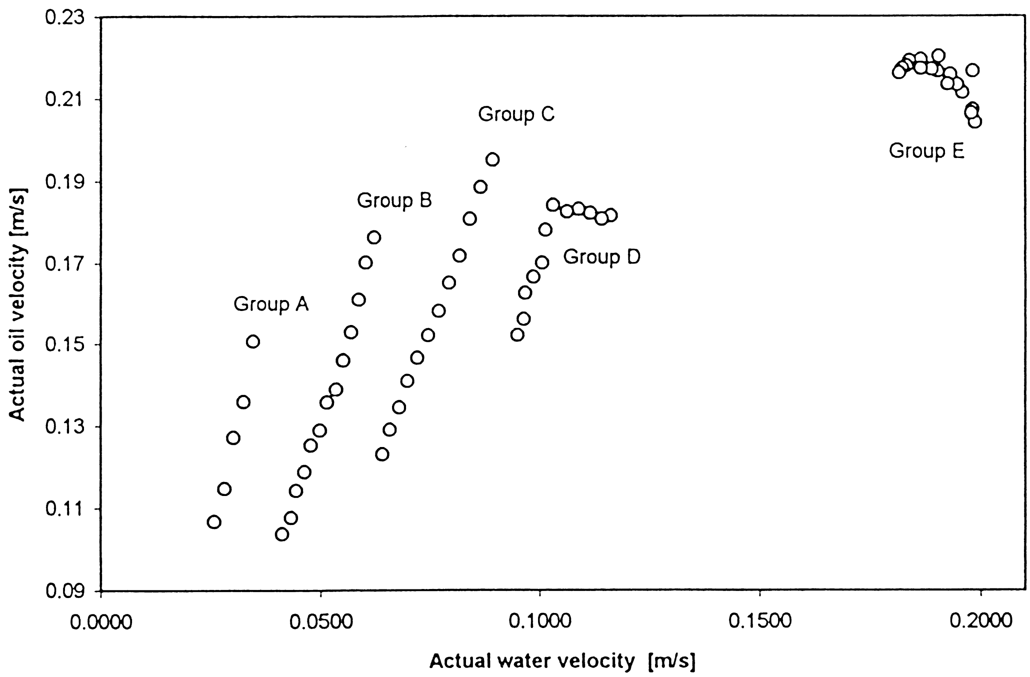

Fig. 6. Measured results, plotted according to the modified drift flux model.

From the steady-state calibration above, it follows that the observed two-phase flow may be approximated over the whole range of measurements by both the original and modified drift flux relationship. However, the precision of such overall calibration is low. Better precision is obtained by calibrating the modified drift flux model along constant mixture velocity lines, as pointed out above. The parameter values obtained by such calibration (Table 2), do not lend to simple physical interpretation, since the additional constraint of constant mixture velocity has been introduced. Nevertheless, the constrained calibration provided more accurate impirical description of the drift-flux behaviour, for the cases considered.

3.3. Transient results and analyses

To investigate water holdup propagation, oil and water were flowed at constant rates, such that steady-state water holdup could be assumed. The input rates were then altered to change the input water holdup. The rate changes were balanced to maintain constant total rate. The induced change of input holdup would propagate up the pipe. The travel time, Δt, between the impedance cells for a given water holdup change was predicted as ratio of distance between holdup measurements, ΔL, and characteristic velocity

$$\Delta t = \frac{\Delta L}{v_c}. \tag{21}$$

The characteristic velocities following from the original and modified drift flux models as a function of steady-state parameters are given by (15) and (18). The steady-state drift flux parameters have been measured as described above. Thus, the travel time of holdup changes between the impedance cells may be predicted using the steady-state drift flux models and compared with actual measurements. This enables comparison of the holdup models, based on their ability to predict propagation, as outlined below.

3.3.1. Propagation of continuity waves

The linear solution describes propagation of small change in water holdup. Fig. 7 illustrates measured propagation compared to estimates by the original drift flux model, (15), and the modified drift flux model, (18). The measured data have been somewhat smoothed, to remove short-term variations. An example of the actual signal can be observed in Fig. 8.

For both the original and the modified drift flux models, the propagation velocity was first calculated using the parameters estimated by linear regression of all steady-state data, as discussed above. The resulting travel time estimates are indicated by square "■" and triangle "▲" in Fig. 7. Obviously, none of these estimates are acceptable.

Secondly, the modified drift flux model was calibrated for the actual mixture flow velocity, Group B data in Fig. 6. The resulting travel time estimate is indicated by dot "•" in Fig. 7, showing close correspondence to measurements. For comparison, the "naive assumption" that disturbances travel at total superficial velocity was also considered. As observed, this over-estimates the travel time, by under-estimating the holdup propagation velocity.

For the higher total superficial velocity of 0.20 m/s, the different models predict continuity wave velocities that are very close to the total superficial and within the errors of

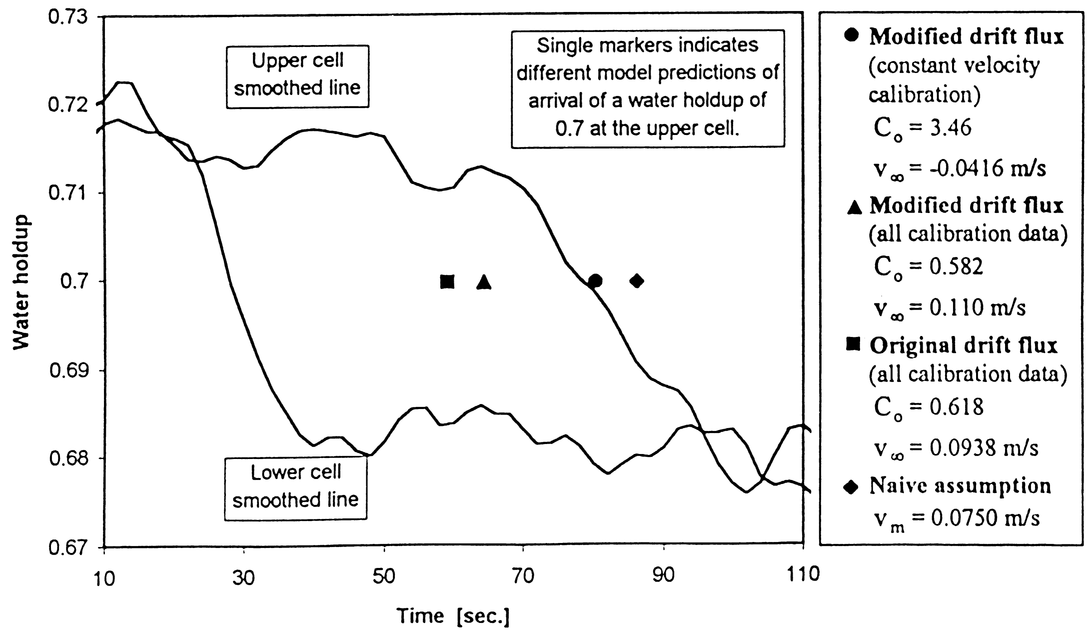

Fig. 7. Propagation of a continuity wave (obtained by reducing input water superficial velocity from 0.037 to 0.034 m/s, while increasing oil superficial velocity from 0.038 to 0.041 m/s).

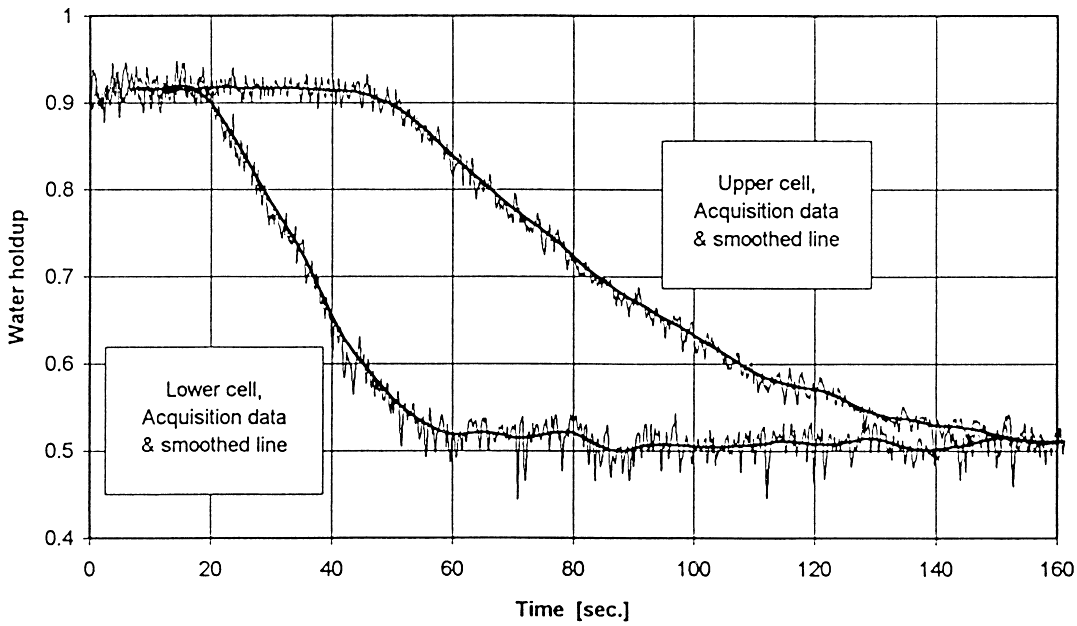

Fig. 8. Measured propagation of a decreasing holdup profile at low rate.

measurements. Discrimination between the different slip flow models, at high flow rate, is therefore not possible by experiments in the relatively short flow loop.

3.3.2. Propagation of decreasing water holdup

Fig. 8 shows the propagation of water holdup decreasing from 0.9 to 0.5. The real acquisition data are plotted together with the averaged profiles. As observed, the profile rarefies significantly between the two cells. The original drift flux model predicts propagation velocity independent of holdup, (15), and thus no distortion. This is in obvious disagreement with measurements. Considering the modified drift flux model with steady-state parameterization for group B data (Table 2), increasing input holdup is predicted to compress, according to (19). Conversely, decreasing input holdup should rarefy, as also is observed from Fig. 8 above.

Fig. 9 shows propagation predicted by different models. The modified drift flux model predicts quite well, when calibrated for the actual mixture flow velocity (Group B data). Acceptable predictions were also obtained for the other low rates (Group A and C data).

Fig. 10 shows holdup propagation at mixture velocity $v_m = 0.20$ m/s. At this rate the oil and water visually appear thoroughly mixed. As observed, the holdup profile does not change much between the upper and lower cell. However, a slight rarefaction and a faster propagation than predicted by the "naive assumption" of homogenous flow is measured. This is most correctly predicted by the modified drift flux model.

The average time shift between the measured profile and the naive assumption is small (approximate 2 sec). This time shift corresponds to approximate 10% difference in input rate if flow behaves as a homogeneous fluid. Given the measurement accuracy of the current facility,

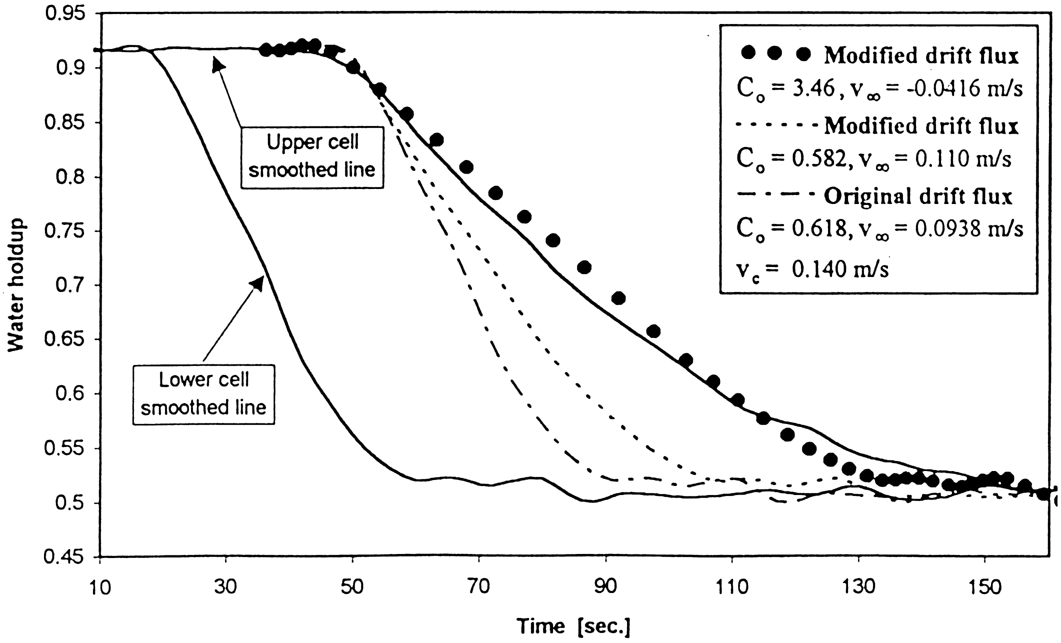

Fig. 9. Propagation of a decreasing holdup profile at low rate (obtained by gradually decreasing input water superficial velocity from 0.055 to 0.022 m/s, while simultaneously increasing oil superficial velocity from 0.020 to 0.053 m/s).

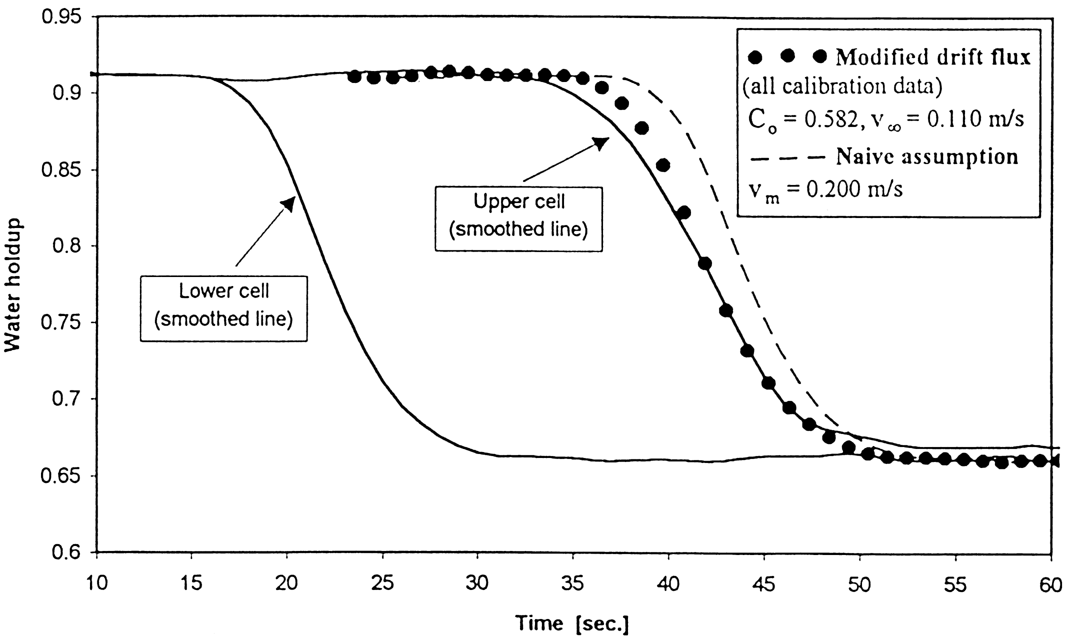

Fig. 10. Propagation of a decreasing holdup profile at higher rate (obtained by gradually decreasing input water superficial velocity from 0.18 to 0.13 m/s, while simultaneously increasing oil superficial velocity from 0.020 to 0.070 m/s, group E data).

the observed differences are considered too small to justify any strong discrimination between the models in this specific case.

3.3.3. Propagation of increasing water holdup

Fig. 11 shows the propagation of a larger increase in water holdup at low flow rate. As observed, the profile compresses between the two cells. Again the modified drift flux model, with parameters obtained by linear regression of flow tests taken at the actual total superficial velocity, predicts water holdup quite well.

Fig. 12 shows the propagation of an initially steeper holdup profile, at same mixture velocity as considered above. In this case the characteristic wave solution using modified drift flux model predicts three different holdups occuring simultaneously at the upper cell at 90 sec. This is of course physically impossible, which means that the hyperbolic model in such cases breaks down as a means of predicting wave propagation. Physicall, the "breaking" is associated with formation and propagation of an abrupt change, or shock (Whitham, 1973). The measured holdup profile shown in Fig. 12 obviously approaches an abrupt change. The propagation of holdup shocks can be predicted by a volume balance accross the shock (Whitham, 1973; Le Veque, 1992). However, this is beyond the scope of the current work.

Fig. 13 shows holdup propagation at higher rate ($v_m = 0.20$ m/s). For the similar experiment illustrated in Fig. 10, the holdup profile does not change much between the cells. However, the same time shift as in Fig. 10 is observed in Fig. 13. This is best predicted by the modified drift

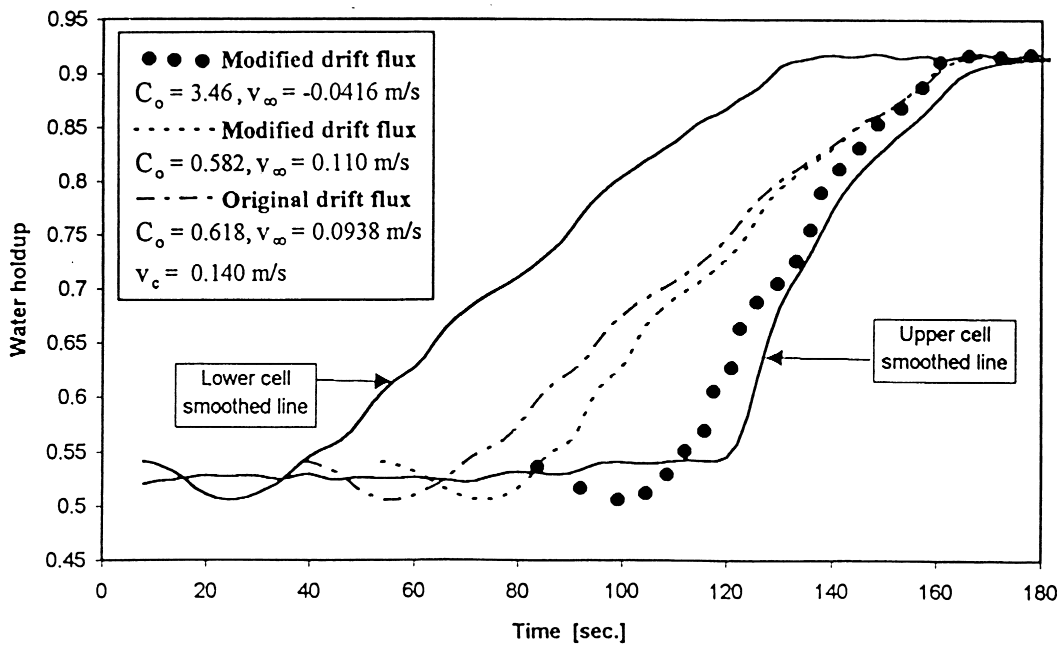

Fig. 11. Propagation of an increasing holdup profile at low rate (obtained by gradually increasing input water superficial velocity from 0.022 to 0.055 m/s, while simultaneously decreasing oil superficial velocity from 0.053 to 0.020 m/s, group B data).

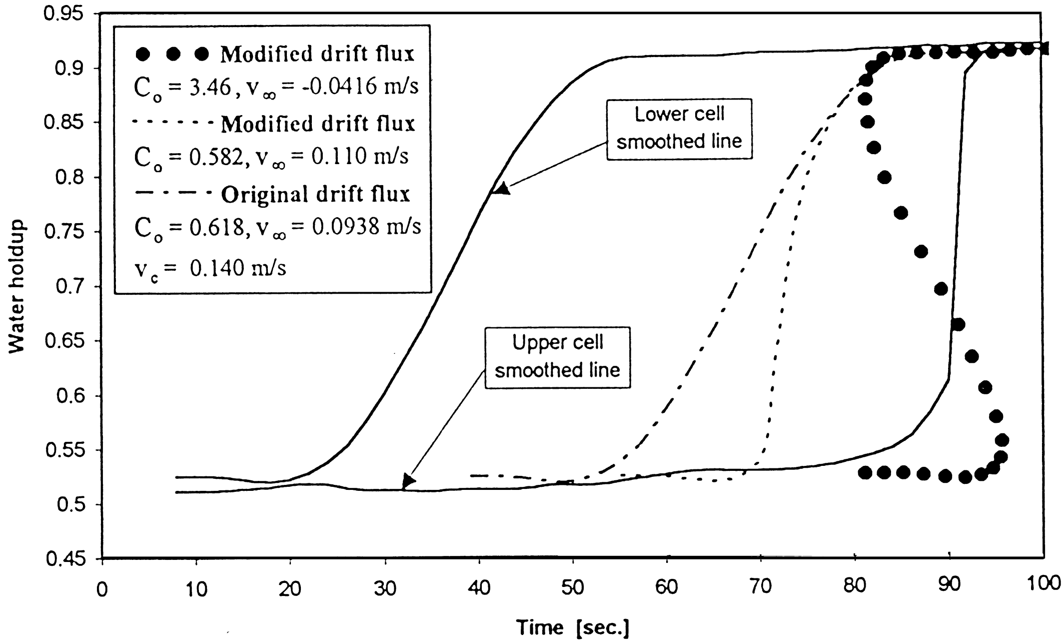

Fig. 12. Propagation of an increasing holdup rofile at low rate (obtained by increasing input water superficial velocity from 0.022 to 0.055 m/s, while simultaneously decresing oil superficial velocity from 0.053 to 0.020 m/s).

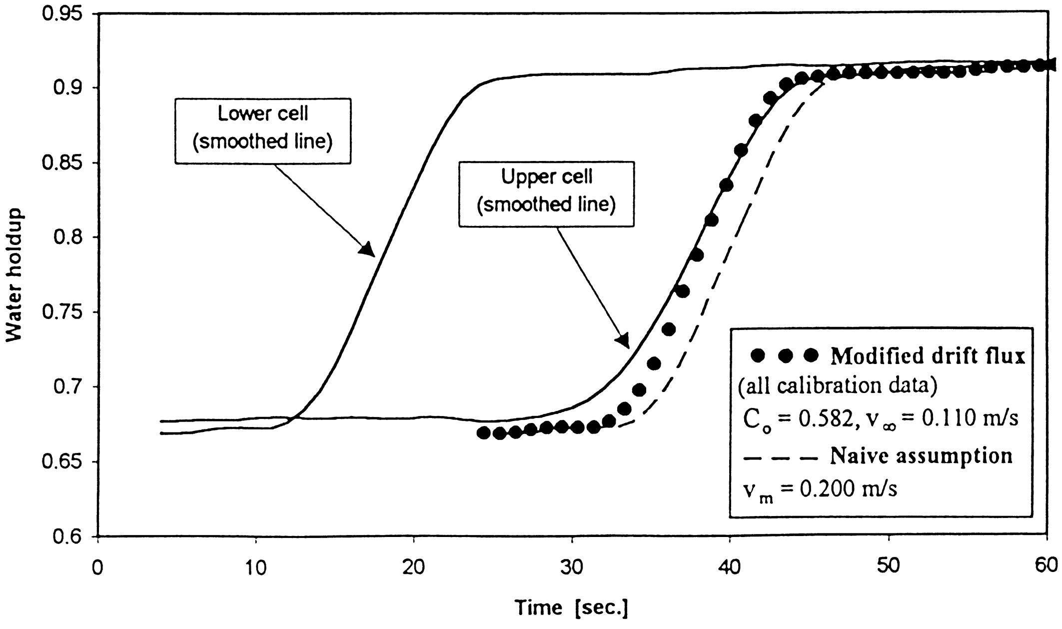

Fig. 13. Propagation of an increasing holdup profile at higher rate (obtained by gradually increasing input water superficial from 0.130 to 0.180 m/s, while simultaneously decreasing oil superficial velocity from 0.070 to 0.020 m/s).

flux model. Again the difference in models predictions are small. Thus, the experimental setup justifies no strong discrimination between models at this higher rate.

4. Discussion

The experimental results above shows that holdup propagation may be predicted quite accurately using a hyperbolic, non-linear wave equation (5) with a sufficiently accurate, steady-state drift flux relationship. Physically, this includes drift, but neglects turbulent diffusion and dispersion. Rarefaction, compression, observed (Figs. 8–10), as predicted by the non-linear hyperbolic wave relation.

In the experiments, the input holdup profile was generated by manipulating the inlet valves. This is likely to also generate short-term distrubances. Short-term disturbances may be expected to propagate at different velocities from the main change, as has been observed by Matuszkiewicz et al. (1987) and by Bouré (1988). Such behaviour is not evident from the current observations. However, our measurements have been smoothed to remove short-periodic variation, as indicated in Fig. 8. It is therefore not unlikely that different methods of analysis also may reveal dispersion.

The flow experiments have been carried out at relatively low velocity and Reynolds number (Table 2). It is likely that turbulent dissipation becomes more significant at higher Reynolds number. However, for the current fluid system, the flow would change from bubbly to emulsified at higher velocity than those considered.

Fischer and Porter variable area flowmeters were used. These were calibrated for both oil and water flow. Due to small oscillations of the floats, a best estimate of the maximum read-off error in total rate is 3%. There is also some error involved when the rates are altered, which is done manually by rotating two wheels simultaneously. The latter error depends on personal skills and the time used to alter the input rates. A perfect rate alteration should produce a smooth profile. As observed in Fig. 11, the measured profile deviates slightly from such a smooth profile. However, it is unlikely that this has great effect on the propagation behaviour. Thus, the total rate error in the transient experiments with two rotameters is expected to be in the range 3–7%. The discrepancy between predictions and measurements at high flow rate may fall within such a range of uncertainty, as discussed above.

5. Conclusions

Holdup propagation has been investigated analytically and experimentally. The following conclusions seem justified for the system considered.

(a) An input holdup profile will distort as it propagates along the pipe. The profile distortions may be predicted using a non-linear, hyperbolic wave propagation model and steady-state drift-flux, or holdup correlations.
(b) The original drift flux model predicts no distortion. It is therefore inconsistent with the measured results.

(c) The modified drift flux model predicts distortion quite well, after the model has been calibrated to describe observed steady-state behaviour.
(d) A generally valid steady-state drift flux, or holdup model with sufficient accurracy to precisely predict propagation remains to be found.

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